Section 4.6
Related Rates

Suppose that a particle is moving along a curve so that its coordinates x and y are differentiable functions of time t. If D is the distance from the origin to the point, then use the Chain Rule to find an equation that relates dD/dt, dx/dt, and dy/dt.

$$D = \sqrt{x^2 + y^2}$$

$$\frac{dD}{dt} = \frac{1}{2}(x^2 + y^2)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

Any equation involving two or more variables that are differentiable functions of time t can be used to find equation that relates their corresponding rates

a.) Assuming that the radius r of a sphere is a differentiable function of t and let V be the volume of the sphere. Find an equation that relates dV/dt and dr/dt

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

b.) Assume that the radius r and height h of a cone are differentiable functions of t and let V be the volume of the cone. Find an equation that relates dV/dt, dr/dt, and dh/dt

$$V = \frac{\pi}{3} r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$

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The radius r, height h, and volume V of a right circular cylinder are related by the equation

$$V = \pi r^2 h$$

a.) How is dV/dt related to dh/dt if r is constant?

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

b.) How is dV/dt related to dr/dt if h is constant?

$$\frac{dV}{dt} = 2\pi rh\frac{dr}{dt}$$

c.) How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?

Strategy for Solving Related Rate Problems

1.) Understand the problem

-Make sure to identify the variable whose rate of change you seek and the variable (variables) whose rate of change you know.

2.) Develop a mathematical model of the problem

Draw a picture and label the parts that are important. Be sure to distinguish constant quantities from variables that change over time.

3.) Write an equation relating the variable whose rate of change you seek with the variable(s) whose rate of change you know.

4.) Differentiate both sides of the equation implicitly with respect to time t.

5.) Substitute values for any quantities that depend on time.

6.) Interpret the solution.

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$$\frac{dr}{dt} = .01$$

$$\frac{dA}{dt} = ?$$

$$r = 50$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(50)(.01) = \pi \, cm^2$$

$$\frac{dl}{dt} = -2\ \text{cm/sec}$$

$$\frac{dw}{dt} = 2\ \text{cm/sec}$$

$$l = 12\ \text{cm}$$

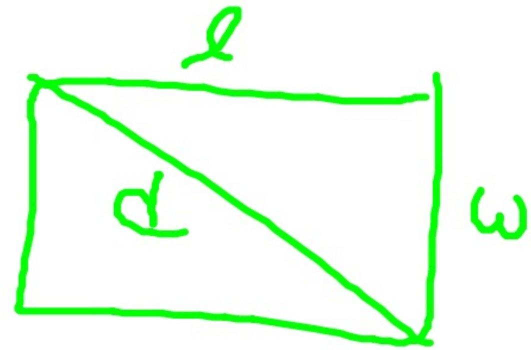
$$w = 5\ \text{cm}$$

$$12^2 + 5^2 = d^2$$

$$144 + 25 = d^2$$

$$169 = d^2$$

$$13 = d$$



$$d^2 = l^2 + w^2$$

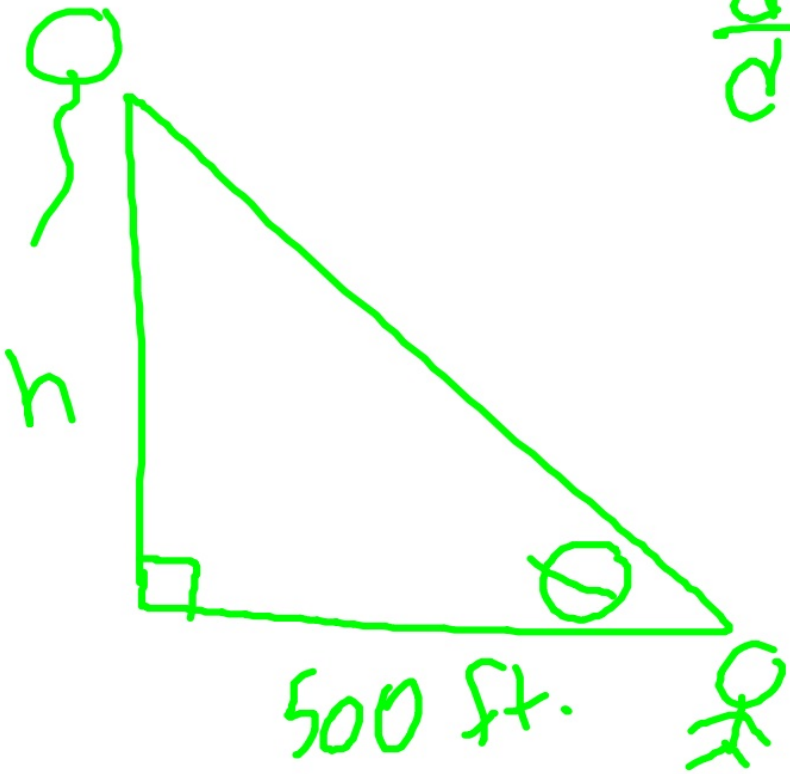
$$2d\frac{dd}{dt} = 2l\frac{dl}{dt} + 2w$$

$$2(13)\frac{dd}{dt} = -48 + 20$$

$$26\frac{dd}{dt} = -28$$

$$\frac{dd}{dt} = -\frac{28}{26} = \frac{-1}{1}$$

A hot-air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at a rate of .14 radians per minute. How fast is the balloon rising at that moment?



$$\frac{dh}{dt} = ?$$

$$\frac{d\theta}{dt} = .14$$

$$\tan\theta = \frac{h}{500}$$

$$h = 500\tan\theta$$

$$\frac{dh}{dt} = 500\sec^2\theta\,\frac{d\theta}{dt}$$

A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is .6 miles north of the intersection and the car is .8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

Water runs into a conical tank at the rate of $9\ ft^3/min$. The tank stands point down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?

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Numbers 8 - 17, 19, 22, 23, 24, 36 - 42